

Couplings of the rho meson in a holographic dual of QCD with Regge trajectories

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Received: 5 January 2008 / Revised version: 10 May 2008 /

Published online: 27 June 2008 – © Springer-Verlag / Società Italiana di Fisica 2008

Abstract. The couplings $g_{\rho HH}$ of the ρ meson with a scalar meson H are calculated in a holographic dual of QCD in which the Regge trajectories for the mesons are manifest. In contrast to the conclusion in general AdS/QCD models, the resulting couplings grow linearly with the quantum number of excited H ; thus they are far from universal. This non-universality seems to result from the disappearance of an explicit cutoff in the holographic dimension in this model. Correspondingly, the ρ -dominance for the electromagnetic form factors of H does not hold any more, even in an apparent sign-alternating manner. With these couplings at hand, the asymptotic behavior of the form factors can easily be calculated. The form factor exhibits the $1/Q^{(2\Delta-2)}$ behavior, which is in accordance with the scaling behavior of the fixed-angle scattering amplitude based on the AdS/CFT correspondence. It is also pointed out that the asymptotic behavior can be matched to the results of perturbative QCD, if the conformal dimension Δ of the operator is replaced by the combination $\tau + L$ of the meson.

PACS. 11.25.Tq; 12.40.Vv; 14.40.Cs

1 Introduction

In 1998, Maldacena advocated the famous duality between string theory on anti-de Sitter (AdS) space and a certain conformal field theory (CFT) on the boundary, now known as the AdS/CFT correspondence [1]. Based on this duality, it was shown by Polchinski that the power law behavior of the high-energy fixed-angle scattering of glueballs in confining gauge theories could be derived non-perturbatively in the dual theory [2]. Since then, various perturbations have been introduced to the original AdS background to produce supergravity duals with a mass gap, confinement, and chiral symmetry breaking, with the purpose of establishing an exact dual of the true QCD. One can introduce a simple cutoff in the radial dimension to simulate confinement and get the so-called hard-wall model [2]. With only one parameter for the cutoff, the lower excited hadron spectrum in this model is found to roughly agree with the physical states [3]. Moreover, chiral symmetry breaking can be modeled rather well in this model too, with two more parameters: the current quark mass and the quark condensate [4–6]. From the top-down approach, one can introduce fundamental quarks by adding D7 branes to the original background, resulting in the D3/D7 model [7], but the resulting theory is asymptotically non-free.

A most successful model, the so-called Sakai–Sugimoto (SS) model [8] was found by introducing D8/D $\bar{8}$ pairs to the supergravity background describing N_c D4 branes compactified on a thermal circle [9]. In this background a confining cutoff appears naturally after compactification. The SS model has been very successful in reproducing many of the qualitative features of non-abelian chiral symmetry breaking in QCD, since spontaneous chiral symmetry breaking appears in a nice geometrical picture. However, in all these models, the masses of highly excited mesons grow linearly with the exciting number, which conflicts with the familiar Regge trajectory [10], where the mass square grows linearly with the exciting number.

The couplings $g_{\rho HH}$ of the ρ meson to a scalar meson H are shown to be quasi-universal in these AdS/QCD models, even in the SS model this universality holds to a good approximation [11]. In [12] it was argued that in generic AdS/QCD models this quasi-universality will hold. Furthermore, the estimation for the quasi-universal coupling nearly equals the expected value m_ρ^2/F_ρ from the vector meson dominance formula. Thus at least an “apparent” ρ -dominance must occur. That means that the individual contribution of excited vector mesons can be of almost the same order as the rho meson, but they must be sign-alternating and compensate with each other, resulting in “accidental” universality. However, since the meson spectrum obtained in these models has a wrong dependence on

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the exciting number, the $g_{\rho HH}$ -universality might be implemented in a wrong way [10].

Linear Regge trajectories for the mesons can be obtained in the dual theory by adding a non-constant dilaton field Φ to the original AdS₅ background [13]. Though ad hoc, the special profile for the dilaton seems to be necessary to guarantee the linear Regge behavior [14]. One can also introduce a gaussian warp factor to the AdS₅ metric, which is shown to be equivalent to the previous background when studying the vector meson spectrum [15]. The linear baryon trajectories were also studied in [16].

In the present paper, we would like to discuss if $g_{\rho HH}$ universality holds in this modified model, which is often referred to as the soft-wall model, or the harmonic oscillator model. The couplings of $g_{\rho HH}$ have been calculated in this model and the results show that the couplings actually grow linearly with the exciting number of H , thus being far from universal. From the quasi-classical picture it seems that the violation of universality is due to lack of an explicit cutoff in the radial dimension. Correspondingly, the ρ -dominance for the electromagnetic form factors of H are strongly violated. The asymptotic behavior of the form factors are also calculated. As expected, the results are in accordance with the scaling behavior of the hard fixed angle amplitude derived on the basis of the AdS/CFT correspondence. However, it seems not to be so direct to match this behavior with the prediction from a perturbative QCD analysis.

This paper is organized as follows. In the next section we give a brief review of the soft-wall model and introduce scalar bulk modes to describe scalar hadron states. In Sect. 3 we give explicit results for $g_{\rho HH}$ and show that $g_{\rho HH}$ universality does not hold any more in this model. Then we go on discussing the asymptotic behavior of the form factors in Sect. 4. The last section is reserved for a summary.

2 Introduction of scalar bulk modes in soft-wall model

2.1 Brief review of soft-wall model

As we have described in the introduction, the soft-wall model is given by introducing a non-constant dilaton field [13] $\Phi(z) = z^2$ to the original AdS metric,

$$ds^2 = e^{2A(z)}(-dz^2 + \eta_{\mu\nu} dx^\mu dx^\nu), \quad (1)$$

with $A(z) = -\log z$. The relevant action for the vector meson is very simple:

$$I = \int d^5x e^{-\Phi(z)} \sqrt{g} \text{Tr} \left\{ -\frac{1}{4g_5^2} (F_L^2 + F_R^2) \right\}, \quad (2)$$

where $A_{L,R} = A_{L,R}^a t^a$, $F_{MN} = \partial_M A_N - \partial_N A_M - i[A_M, A_N]$ and $t^a = \sigma^a/2$. The gauge coupling g_5 can be fixed by matching the UV asymptotics of the current-current two-point function between bulk and boundary theories, $g_5^2 = 12\pi^2/N_c$. The vector mesons (ρ , etc.) are considered to be

dual to the normalizable modes of the vectorial combination $V = (A_L + A_R)/2$. The non-normalizable mode of V is considered to be dual to the corresponding conserved current in the 4d theory and in the U(1) case just the electromagnetic current J_μ^{em} .

We can use the gauge invariance of the action to go to the axial gauge, $V_z = 0$, and derive the equation for the 4d-transverse components V_μ^T ($\partial^\mu V_\mu^T = 0$). It has normalizable solutions, v_n , only for discrete values of the 4d momentum q^2 equal to m_n^2 :

$$\partial_z(e^{-B}\partial_z v_n) + m_n^2 e^{-B} v_n = 0, \quad (3)$$

where $B = \Phi(z) - A(z)$. By making the substitution

$$v_n = e^{B/2} \psi_n, \quad (4)$$

this equation can be brought into the form of a Schrödinger equation:

$$-\psi_n'' + V(z)\psi_n = m_n^2 \psi_n, \quad (5)$$

$$V(z) = (B')^2/4 - B''/2 = z^2 + 3/(4z^2). \quad (6)$$

Thus we can easily read off the squared masses of the ρ s:

$$m_n^2 = 4(n+1), \quad (7)$$

which exhibits the right Regge behavior. The corresponding normalized eigenfunctions are

$$\psi_n(z) = e^{-z^2/2} z^{3/2} \sqrt{\frac{2}{n+1}} L_n^1(z^2), \quad (8)$$

where L_n^1 are the associated Laguerre polynomials. The original mode functions v_n are given by

$$v_n(z) = z^2 \sqrt{\frac{2}{n+1}} L_n^1(z^2), \quad (9)$$

from which the corresponding decay constants can be obtained by considering the two-point correlation function [4–6],

$$F_{\rho_n}^2 = \frac{1}{g_5^2} [v_n''(0)]^2 = \frac{8(n+1)}{g_5^2}. \quad (10)$$

On the other hand, the non-normalizable mode of V can be expressed by $V_\mu(q, z) = V(q, z)V_\mu^0(q)$, where $V_\mu^0(q)$ is the electromagnetic source and $V(q, z)$ is the bulk-to-boundary propagator. Then $V(q, z)$ satisfies the following equation:

$$\partial_z(e^{-B}\partial_z V) + q^2 e^{-B} V = 0, \quad (11)$$

with $q^2 < 0$. $V(q, z)$ must also satisfy the boundary condition

$$V(q, z=0) = \text{const} \quad (12)$$

to ensure the interpretation of $V_0^\mu(q)$ as the electromagnetic source. For convenience in discussing the form factor,

this constant can be chosen to be $1/g_5$. Then the solution of $V(q, z)$ can be found [17]:

$$V(q, z) = \frac{1}{g_5} \Gamma(1 - q^2/4) U(-q^2/4, 0z^2), \quad (13)$$

with $U(a, b, c)$ the confluent hypergeometric function. It can also be written as [18]

$$V(q, z) = -\frac{q^2}{4g_5} \int_0^1 dx x^{-q^2/4-1} \exp\left[-\frac{x}{1-x} z^2\right], \quad (14)$$

from which it follows that, if $q^2 = 0$, then $V(0, z) = 1/g_5$. An important fact is that the non-normalizable mode can always be decomposed into the normalizable modes [19], and in this model this relation is given by [18]

$$V(q, z) = \sum_n \frac{F_{\rho_n} v_n(z)}{-q^2 + m_n^2}. \quad (15)$$

2.2 Introduction of scalar bulk modes

Now we will introduce scalar states into this model along the lines as in [19]. A similar discussion on the scalar glueball spectrum has been made in [20]. The action is just that of a free massive scalar field in the above background:

$$I[\phi] = \int d^5x e^{-\Phi(z)} \sqrt{g} \left\{ \frac{1}{2} ((\nabla\phi)^2 - m_5^2 \phi^2) \right\}. \quad (16)$$

The hadronic states H are considered to be created by the operator dual to the normalized modes of ϕ . The AdS mass is given by the duality relation,

$$m_5^2 = \Delta(\Delta - 4), \quad (17)$$

where Δ is the conformal dimension of the operator. The equation of motion reads from the action

$$\partial_z (e^{-B_1} \partial_z \phi_H) + m_H^2 e^{-B_1} \phi_H - \frac{m_5^2}{z^2} e^{-B_1} \phi_H = 0, \quad (18)$$

with $B_1(z) = \Phi(z) - 3A(z)$. By applying the Bogoliubov transformation, $\psi_H(z) = e^{-B_1(z)/2} \phi_H(z)$, the function $\psi_H(z)$ satisfies the one dimensional Schrödinger equation:

$$-\psi_H''(z) + V_H(z) \psi_H(z) = m_H^2 \psi_H(z). \quad (19)$$

The potential $V_H(z)$ is given by

$$V_H(z) = z^2 + 15/4z^2 + 2 + m_5^2/z^2. \quad (20)$$

Then one can easily get the eigenfunctions

$$\psi_a^\Delta(z) = e^{-z^2/2} z^{\Delta-3/2} \sqrt{\frac{2a!}{(\Delta+a-2)!}} L_a^{\Delta-2}(z^2), \quad (21)$$

and the corresponding eigenvalues are $m^2 = 4a + 2\Delta$. Doing the Bogoliubov transformation again we get the original mode function:

$$\phi_a^\Delta(z) = z^\Delta \sqrt{\frac{2a!}{(\Delta+a-2)!}} L_a^{\Delta-2}(z^2), \quad (22)$$

which is normalized as

$$\int \frac{dz}{z^3} e^{-\Phi(z)} \phi_a^\Delta(z)^2 = 1. \quad (23)$$

Certainly $\phi_a^\Delta(z) \rightarrow z^\Delta$ as $z \rightarrow 0$, as it should.

3 The ρ couplings $g_{\rho HH}$

The hadronic matrix element for the electromagnetic current in our background has the form [21]

$$ig_5 \int d^4x dz e^{-\Phi(z)} \sqrt{g} V^M(x, z) \Phi_{P'}^*(x, z) \overleftrightarrow{\partial}^M \Phi_P(x, z), \quad (24)$$

with $M = 0, 1, 2, 3, z$. The electromagnetic current corresponds to excitation of the non-normalizable modes of $V_M(x, z)$ along Minkowski coordinates, $V_\mu(q, z) = V_\mu^0(q) V(q, z) = \epsilon_\mu e^{iq \cdot x} V(q, z)$. The incoming (outgoing) hadron is represented by $\Phi_P(x, z) (\Phi_{P'}^*(x, z))$, which is plane wave along the Poincaré coordinates, $\Phi_P(x, z) = e^{iP \cdot x} \phi_H(z)$ with $P_\mu P^\mu = m_H^2$ giving the mass of the hadron. Thus the integration over the Minkowski coordinates can be done and we get the form factor

$$F_H(Q^2 = -q^2) = g_5 \int \frac{dz}{z^3} e^{-\Phi(z)} V(q, z) \phi_H(z)^2. \quad (25)$$

The normalization of ϕ_H and the boundary condition of $V(q, z)$ ensure that $F_H(0) = 1$. According to (15) the form factor can also be decomposed as

$$F_H(Q^2) = g_5 \int \frac{dz}{z^3} e^{-\Phi(z)} \sum_n \frac{F_{\rho_n} v_n(z)}{Q^2 + m_n^2} \phi_H(z)^2, \quad (26)$$

from which we get the couplings $g_{\rho_n HH}$:

$$g_{\rho_n HH} = g_5 \int \frac{dz}{z^3} e^{-\Phi(z)} v_n(z) \phi_H(z)^2. \quad (27)$$

Now we would like to calculate the ρ couplings $g_{\rho HH}$ to the scalar states H as a function of the excitation level a and the conformal dimension Δ of the corresponding operator, as in [12]. The resulting couplings are

$$F_\rho g_{\rho aa}^{(3)} / m_\rho^2 = 2, 4, 6, 8, 10, \dots, \quad (28)$$

for $a = 0, 1, 2, 3, 4, \dots$, and

$$F_\rho g_{\rho 00}^{(\Delta)} / m_\rho^2 = 2, 3, 4, 5, 6, \dots, \quad (29)$$

for $\Delta = 3, 4, 5, 6, 7, \dots$. It is shown that $g_{\rho HH}$ actually grows linearly with the exciting number a and the conformal dimension Δ . This is quite different from the behavior of $g_{\rho HH}$ in the hard-wall and the D3/D7 model considered in [12], where the $g_{\rho HH}$ are shown to be quasi-universal and to lie within a narrow band near m_ρ^2 / F_ρ .

To make this more clear, let us analyze the integral in (27) carefully. As shown in [12], the $g_{\rho HH}$ -universality in

generic AdS/QCD models is based on two facts. First, the radial dimension must terminate at $z = z_{\max} \sim 1/\Lambda_{\text{QCD}}$ to get confinement in the infrared; second, being the lowest mode of a conserved current, the ρ meson should be structureless and have no nodes, and it must satisfy Neumann boundary conditions at $z = z_{\max}$. In other words, $v_0(z)$ will always increase monotonously with z and reach its maximal value at $z = z_{\max}$. Thus the integral in (27) (with $\Phi(z) = 0$ in generic AdS/QCD models) is always dominated by the contributions in the region $z \sim z_{\max}$, where $v_0(z)$ varies slowly and has a typical value $v_0(z_{\max})$. Replacing $v_0(z)$ by $v_0(z_{\max})$ in (27) and using the normalization condition for $\phi_H(z)$, we finally get the universal coupling $g_{\rho HH}$ and can actually prove that it equals the expected value m_ρ^2/f_ρ from ρ -dominance [12]. As a result, there must be at least apparent ρ -dominance to ensure this quasi-universality. However, in the soft-wall model, there is not an absolute cutoff z_{\max} . Although the dilaton factor formally introduces a “soft” wall, the space never terminates. Thus various modes can actually extend anywhere in this dimension. This can be seen from Fig. 1, where the probability density $D_a^\Delta(z) = e^{-\Phi(z)} \phi_a^\Delta(z)^2/z^3$ for the first three radial states of the $\Delta = 3$ meson have been plotted. The extension size then sets an effective cutoff in the integral (27), and we can estimate the integral in a similar manner as in [12]. Based on the quasi-classical arguments in [10], one may expect that the size of the extension should be proportional to the length $L_a \sim M_a/\sigma$ of the flux tube of the excited meson, where σ is the tension of the flux tube. Note that $v_0(z) = \sqrt{2}z^2$ and we have a linear relation of mass square, $M_a^2 \sim a$; one should then replace $v_0(z)$ by $z^2 \sim L_a^2 \sim a$. Thus one gets the linearly increasing coupling $g_{\rho HH}$.

It is well known that ρ meson coupling universality can be induced from the vector meson dominance hypothesis; that is, the ρ meson gives the dominant contribution to the electromagnetic form factor of the various hadrons.

Now since $g_{\rho HH}$ -universality does not hold any more, we can conclude that ρ meson dominance must be violated at the same time. To check this, we proceed to calculate the couplings of the excited ρ mesons with H . The calculation is straightforward; we just need to replace the ρ mode in (27) by the corresponding exciting ρ states. For the radial exciting states, the results are as follows: for $n = 0, 1, 2, 3, 4, \dots$,

$$\begin{aligned} F_{\rho n} g_{\rho n 00}^{(3)}/m_{\rho n}^2 &= 2, -1, 0, 0, \dots, \\ F_{\rho n} g_{\rho n 11}^{(3)}/m_{\rho n}^2 &= 4, -8, 8, -3, 0, 0, \dots, \\ F_{\rho n} g_{\rho n 22}^{(3)}/m_{\rho n}^2 &= 6, -21, 44, -54, 36, -10, 0, 0, \dots \end{aligned} \quad (30)$$

For the states described by $\Delta = 3, 4, 5, \dots$, we have

$$\begin{aligned} F_{\rho n} g_{\rho n 00}^{(3)}/m_{\rho n}^2 &= 2, -1, 0, 0, \dots, \\ F_{\rho n} g_{\rho n 00}^{(4)}/m_{\rho n}^2 &= 3, -3, 1, 0, 0, \dots, \\ F_{\rho n} g_{\rho n 00}^{(5)}/m_{\rho n}^2 &= 4, -6, 4, -1, 0, 0, \dots, \end{aligned} \quad (31)$$

with all the neglected terms vanishing. We see that there is no evidence of vector meson dominance in the form factors of these states. Though the couplings indeed show a sign-alternating pattern, they do not compensate with each other to give even apparent ρ -dominance.

4 Asymptotic behavior of the electromagnetic form factor

With all these couplings at hand, the asymptotic behavior of the form factor can be calculated easily using (26). The results are radial number dependent, and we give the

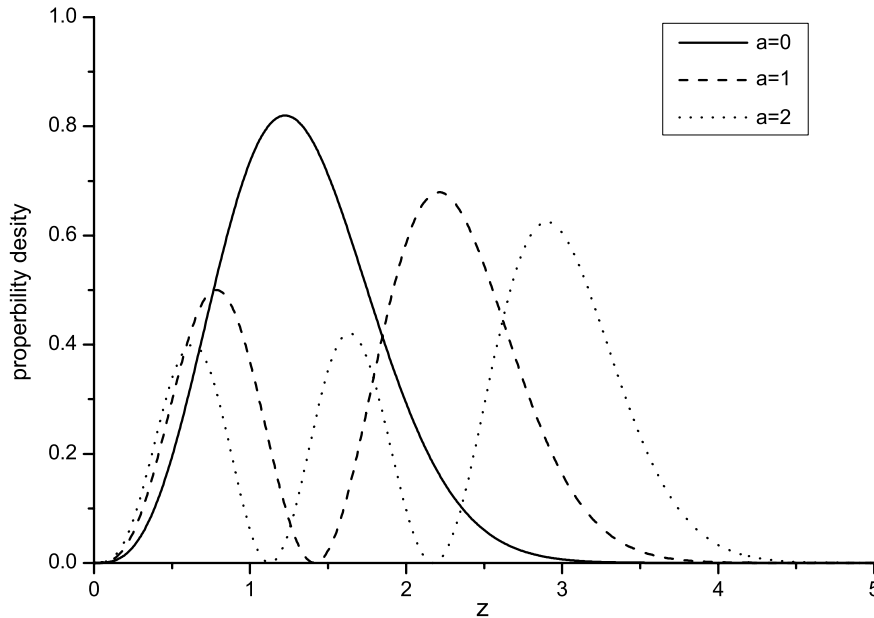


Fig. 1. Probability density $D_a^\Delta(z)$ of the first three radial states of the $\Delta = 3$ meson

results for the first few states only:

$$\begin{aligned}
F_{00}^{(\Delta=3)}(Q^2) &\rightarrow (\Delta-1) \frac{m_\rho^2}{Q^2} \left(\frac{m_\rho^2}{Q^2} \right), \\
F_{11}^{(\Delta=3)}(Q^2) &\rightarrow (\Delta-1) \frac{m_\rho^2}{Q^2} \left(\frac{m_{\rho_1}^2}{Q^2} \right), \\
F_{22}^{(\Delta=3)}(Q^2) &\rightarrow (\Delta-1) \frac{m_\rho^2}{Q^2} \left(\frac{m_{\rho_2}^2}{Q^2} \right), \\
F_{00}^{(\Delta=4)}(Q^2) &\rightarrow (\Delta-1) \frac{m_\rho^2}{Q^2} \left(\frac{m_\rho^2}{Q^2} \frac{m_{\rho_1}^2}{Q^2} \right), \\
F_{11}^{(\Delta=4)}(Q^2) &\rightarrow (\Delta-1) \frac{m_\rho^2}{Q^2} \left(\frac{m_{\rho_1}^2}{Q^2} \frac{m_{\rho_2}^2}{Q^2} \right), \\
F_{22}^{(\Delta=4)}(Q^2) &\rightarrow (\Delta-1) \frac{m_\rho^2}{Q^2} \left(\frac{m_{\rho_2}^2}{Q^2} \frac{m_{\rho_3}^2}{Q^2} \right). \quad (32)
\end{aligned}$$

From the above relations one can guess the asymptotic behavior for generic states with radial number a and arbitrary dimension Δ :

$$F_{aa}^{(\Delta)}(Q^2) \rightarrow (\Delta-1) \frac{m_\rho^2}{Q^2} \left(\prod_{k=a}^{k=a+\Delta-3} \frac{m_{\rho_k}^2}{Q^2} \right). \quad (33)$$

Using $m_{\rho_k}^2 = 4k + 4$ this can be simplified to

$$F_{aa}^{(\Delta)}(Q^2) \rightarrow (\Delta-1) \frac{(a+\Delta-2)!}{a!} \left(\frac{m_\rho^2}{Q^2} \right)^{\Delta-1}. \quad (34)$$

For $a = 0$ this reduces to

$$F_{00}^{(\Delta)}(Q^2) \rightarrow (\Delta-1)! \left(\frac{m_\rho^2}{Q^2} \right)^{\Delta-1}, \quad (35)$$

which has been derived analytically in [17].

The above relations exhibit the $1/Q^{(2\Delta-2)}$ behavior, which is in accordance with the scaling behavior of the hard fixed angle scattering derived in [2]. These relations can easily be generalized to the pseudo-scalar and vector mesons, and the resulting form factor will have the same asymptotic behavior. However, from the perturbative QCD analysis one may expect a $1/Q^{(2(\tau+L)-2)}$ behavior [22], where the twist τ is defined by $\tau = \Delta - \sigma$ with σ the sum over the constituents' spin $\sigma = \sum_{i=1}^n \sigma_i$; L is the angular momentum. The twist is also equal to the number of partons $\tau = n$. In the case of the scalar mesons considered, this does not make any difference since we have $\Delta = \tau + L$. But for the pseudo-scalar and vector mesons, this does not hold any more. To eliminate this discrepancy, [23] suggests that we substitute $\tau + L$ instead of Δ in the AdS mass relation (17). Indeed the form factor of the pseudo-scalar mesons can be matched to the perturbative QCD results after doing so, but there is still some problem in treating the vector mesons [17]. The main difficulty may be described as follows.

On the AdS/QCD side, the hadron is treated entirely without referring to the parton degree of freedom, so the form factor scales according to the conformal dimension of

the hadron operator. But in the perturbative QCD analysis, we get the asymptotic behavior related to the number of partons. To successfully match the results of both sides, one should try to clarify the physical meaning of replacing the conformal dimension Δ by the combination $\tau + L$. This subject will be considered in the future.

5 Summary

In this paper we employ the AdS/QCD models with linear Regge trajectories and calculate the couplings $g_{\rho HH}$ in this typical holographic model, which is often referred to as the soft-wall model or the harmonic oscillator model. The calculated result of the couplings $g_{\rho HH}$ shows that the $g_{\rho HH}$ grow linearly with the radial number a and the operator dimension Δ of H and thus are far from universal. The key point may be that there is not an explicit cutoff in this model. As an inference, the vector meson dominance does not hold any more, even in an apparent sign-alternating manner. The asymptotic behavior of the form factor for various states is studied and we find accordance with the expectation from the general derivation based on gauge/string duality. It seems possible to match this behavior to perturbative QCD results if we replace the conformal dimension Δ by the combination $\tau + L$ of the meson. However, the physical meaning of this replacement still needs further investigation.

It is interesting to note that linear Regge trajectories can also be obtained by doing a classical string calculation in confining backgrounds, in which the spinning string is almost located in the holographic dimension [24]. One may expect that ρ coupling universality and further ρ -dominance will be implemented in these models in a way consistent with linear Regge trajectories.

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